Resonant charge and spin transport in a $\mathcal{T}$-stub coupled to a superconductor

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Abstract. - We study transport through a single channel $\mathcal{T}$-stub geometry strongly coupled to a superconducting reservoir. In contrast to the standard stub geometry which has both transmission resonances and anti-resonances in the coherent limit, we find that due to the proximity effect, this geometry shows neither a $T = 1$ resonance (T is the transmission probability for electrons incident on the $\mathcal{T}$-stub) nor a $T = 0$ anti-resonance as we vary the energy of the incident electron. Instead, we find that there is only one resonant value at $T = 1/4$, where charge transport vanishes while the spin transport is perfect.

Introduction. – One of the many intriguing issues in the subject of spintronics [1] concerns production and detection of pure spin current. A simple minded but popular proposal for production of pure spin current (SC) involves electrons flowing with equal flux in opposite directions with opposite spin polarizations. This situation results in the exact cancellation of the charge current while the spin current adds up. Alternatively one can also produce pure SC by having a unidirectional flow of electrons and holes together with equal flux and spin polarization. In this case also, the charge current cancels out leaving behind a pure spin current. In this paper we will work along the lines of the second proposal for production of pure SC .

Situations involving current carried by an admixture of electrons and holes are naturally realized in systems involving superconductor-normal junctions [2–9] due to the interplay of Andreev reflection and normal reflection at the interface between the superconductor and the normal metal. A recent proposal by the present authors which exploited this fact for production of pure SC involved transport of electrons and holes across a normal-superconducting-normal (NSN) junction [10]. The normal metals in the NSN junction were considered to be one-dimensional interacting electron gases which were modelled as Luttinger liquid wires, and a situation corresponding to pure spin current was shown to be an unstable fixed point of the theory. In contrast to our previous study, here we work with free electrons and pure spin current is produced due to resonant conversion of incident electrons into transmitted electrons or holes with equal amplitude across a $\mathcal{T}$-stub geometry which is strongly coupled to a superconducting reservoir. Inclusion of electron-electron interaction can lead to very interesting physics [11] in the presence of resonances but this is beyond the scope of the present work.

Theoretically, although our model appears simple, it is one of the first models to realize resonant transmission of electrons through a complex barrier (in this case a stub, which hosts both electron and hole waves due to its coupling to superconductor at one end) with an amplitude which is not unimodular. To develop an understanding of the new resonance, we explore the analytic structure of the electron transmission amplitude in the complex energy plane. We show that the analytic structure of the transmission function is quite different from the cases of standard double barrier resonances or the resonance-anti resonance pairs of the normal stub geometry [12, 13]. In this article, we consider a single mode $\mathcal{T}$-stub which is coupled to a superconducting reservoir at one end, and to a single mode wire at the other end, which is then connected to the left and right reservoirs (see Fig.1). We assume that the current injected from the left reservoir into the wire is completely spin polarized. Additionally we assume that the bias which drives current between the

left and the right reservoir is such that the net current is always flowing from left to right. Even though our calculations use a single mode approximation, these results can be applied to a wire with multiple channels as long as the scattering matrix at the junction does not mix the channels substantially at the junction - *i.e.*, as long as the S-matrix is block diagonal, in each of the channels, or can be approximated as being almost block diagonal. In the general case, with substantial inter-mode scattering at the junction, detailed numerical analysis, beyond the scope of this analysis, would be required to compute the transmission function.

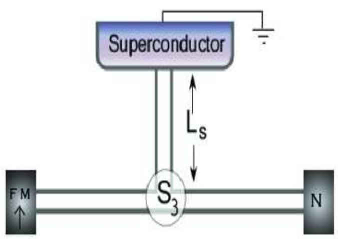

Fig. 1: Cartoon of the set-up proposed for the $\mathcal{T}$-stub geometry. L_s is the length of the stub and $\mathbb{S}_3$ represents the $\mathbb{S}$-matrix describing the three-wire junction.

Here the role of the bulk superconductor is to convert an incoming electron in the stub into an outgoing hole via Andreev reflection (AR). We restrict ourselves to the case where the coupling between the superconducting reservoir and the stub is perfect, so that the AR probability of an incident electron to reflect as a hole is unity.

We find that as we vary the energy of the incident electrons for fixed length of the stub or the length of the stub for fixed incident energy, due to the proximity effect we get a resonance at $T = 1/4$ where the charge current across the stub is identically zero while spin current adds up. Thus, we get resonant transport of pure SC, which is exactly matched by the "anti-resonant" transport of the total charge current. The advantage of this set-up is that the detection of the zero total charge current across the $\mathcal{T}$-stub automatically ensures simultaneous detection of pure spin current. Note that because of the perfect conversion of electrons to holes (and vice-versa) at the boundary of the stub and the superconductor, the scattering matrix at the three wire junction (describing transport between lead-1 and lead-2, see Fig. 1) can be parameterized by an effective 4×4 $\mathbb{S}$-matrix. This includes both the particle and the hole sector. Transport across the junction involves two new processes other than the usual reflection and transmission of electrons (holes) - *(a)* transmission of an incident electron state into a hole state across the stub, which is usually called crossed Andreev reflection (CAR) and *(b)* reflection of an incident electron state into a hole state from the stub (defined earlier as AR). These processes result from the fact that an electron incident on the junction can tunnel into the stub branch and bounce back to the junction as a hole. In fact, it can bounce back and forth between the superconductor and the three wire junction undergoing multiple electron-hole conversions. If the incident electron undergoes an even number of bounces, it will come out as an electron but if it undergoes an odd number of bounces, it will come out as a hole. Hence when the electron exits the stub after an odd number of bounces, it contributes to the electron-hole amplitudes. If it exits the stub branch as a hole on the same wire, from where it entered, it will contribute to the AR and if it exits on the other wire, it will contribute to the CAR. Hence, even though the wires-1,2 (coupled to the reservoir-1,2, see Fig. 1) are not directly coupled to the superconductor and there are no Andreev processes at the three wire junction, the stub allows for finite amplitudes for processes which effectively mimic CAR process between wire-1 and wire-2. At low temperatures ($L_{th} > L_s$, where L_{th} is the thermal length and L_s is the stub length) the stub branch acts like a coherent particle-hole resonator leading to resonances in the amplitude for the effective CAR process and the direct electron transmission process, due to quantum interference between the particle and hole waves inside the stub. *The resonant production of electrons and holes in wire-2 in response to incident electrons from wire-1 on the $\mathcal{T}$-stub, due to quantum interference between particle and hole waves inside the stub, can lead to resonant production of pure SC in wire-2. This is the main result of this paper.*

Quantum mechanics of the $\mathcal{T}$-stub. – We start with a junction of three wires, with two of them connected to a fully spin polarized reservoir and a normal reservoir respectively (see Fig.1) and the third one (a stub of finite length L_s) coupled to a superconducting reservoir. A normal stub would have boundary conditions at the stub end, which would completely reflect an incoming electron to an outgoing electron. Here, on the other hand, the superconducting reservoir turns an incoming electron completely to an outgoing hole in the perfect Andreev limit. The 3×3 $\mathbb{S}$-matrix coupling the wires to the stub initially is a 'normal' scattering matrix for electrons or holes with spin-up ($\uparrow$) or spin-down ($\downarrow$) and left-right symmetry. We have considered the $\mathbb{S}$-matrix to be the same for the electron and hole sector assuming electron-hole symmetry. As studied earlier [14], the $\mathbb{S}$-matrix for a three-wire junction can be described by a single parameter, if we choose real parametrization. In terms of the single parameter r', the $\mathbb{S}$-matrix describing electrons ($\uparrow$ / $\downarrow$) or holes ($\uparrow$ / $\downarrow$) is given by

$$\mathbb{S}_3 = \begin{bmatrix} r' & t' & t \\ t' & r' & t \\ t & t & r \end{bmatrix}, \qquad (1)$$

where using unitarity, we have

$$t' = 1 + r', r = -1 - 2r', t = \sqrt{(-2r')(1 + r')}, \qquad (2)$$

and $-1 \leqslant r' \leqslant 0$. Here, we consider the single parameter r' to denote the degreee of coupling between the wires 1

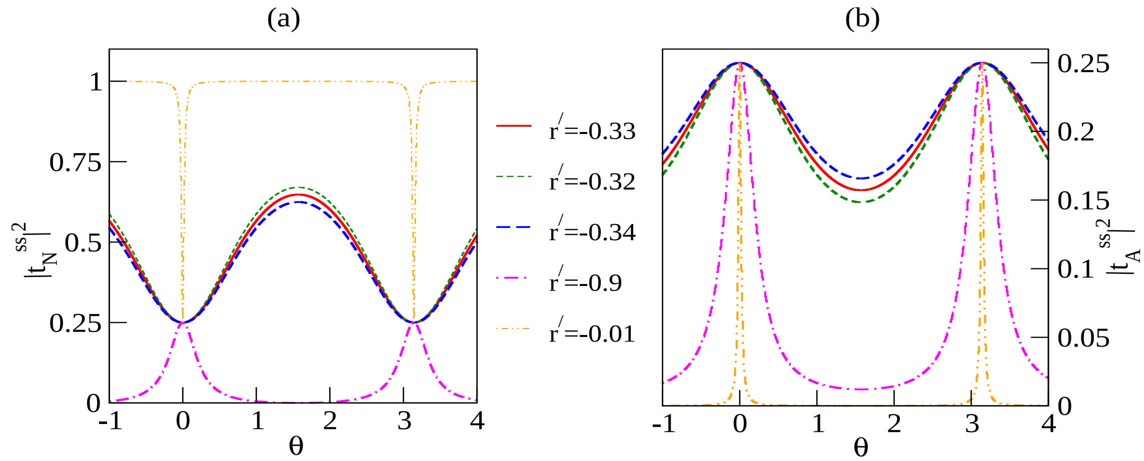

Fig. 2: (Color online)(a) Transmission probability $|t_N^{ss}|^2$ and (b) CAR probability $|t_A^{ss}|^2$ are plotted in units of e^2/h as a function of $\theta = 2EL_s/\hbar v_F - \cos^{-1}(E/\Delta)$ for five different values of r'.

and 2 and the stub. For $r' = -1$, both the wires and the stub are all disconnected from each other. For $r' = 0$, the two wires are fully connected to one another, but disconnected from the stub. At $r' = -0.5$, $r = 0$, which means that multiple reflections within the stub do not occur, although the wire is well-coupled to the stub. Another interesting value for r' is $r' = -1/3$ as this corresponds to the most symmetric $\mathbb{S}$-matrix describing the junction. The other end of the stub is assumed to be perfectly coupled to a superconducting reservoir, and the 2×2 $\mathbb{S}$-matrix describing the superconducting boundary is given by

$$\mathbb{S}_{\text{bound}} = \begin{bmatrix} r^s & r_a^s \\ r_a^s & r^s \end{bmatrix} , \tag{3}$$

which, in the perfect Andreev limit [15] has $r_a^s = \alpha = e^{-i\cos^{-1}(E/\Delta)} \times e^{\pm i\phi}$ and $r^s = 0$. For $E = 0$, α reduces to $-ie^{\pm i\phi}$ where ϕ is an energy independent part. When the AR is not perfect, *i.e.* $r^s \neq 0$.

Now, let us compute the effective 4×4 $\mathbb{S}$-matrix coupling wire-1 and wire-2. As discussed in the introduction, since the stub hosts both electrons and holes, the junction of wire-1 and wire-2 effectively has a $\mathbb{S}$-matrix very similar to a normal-superconducting-normal (NSN) junction. The coherent amplitudes for the reflection, transmission, AR and CAR can be calculated by adding up all possible Feynman paths taken by the incident electron while bouncing back and forth inside the $\mathcal{T}$-stub. These amplitudes depend only on a single parameter r' which parameterize the 3×3 $\mathbb{S}$-matrix at the junction and are given by

$$r_N^{ss} = r' + \frac{\alpha^2 r t^2 \eta^2}{(1-\alpha^2 r^2 \eta^2)} , \tag{4}$$

$$t_N^{ss} = t' + \frac{\alpha^2 r t^2 \eta^2}{(1-\alpha^2 r^2 \eta^2)} , \tag{5}$$

$$r_A^{ss} = t_A^{ss} = \frac{\alpha t^2 \eta e^{i\phi}}{(1-\alpha^2 r^2 \eta^2)} , \tag{6}$$

where r_N^{ss}, t_N^{ss}, r_A^{ss}, t_A^{ss} are the reflection, transmission, AR and CAR amplitudes respectively. The superscript ss stands for the case of superconducting stub where the stub is strongly coupled to the superconductor while the subscript N or A corresponds to normal or Andreev processes. The r, t and t' were defined earlier in terms of r' in Eq. 2. Note the fact that the amplitudes for the AR and CAR are the same owing to the left-right symmetry of the S-matrix representing the junction. The energy dependent phase η comes from adding the paths of the incident electron, which gets converted to a hole at the superconducting boundary, at the first bounce and all odd bounces and back to an electron at all even bounces. Here $\eta = e^{i2EL_s/\hbar v_F}$ and E is the energy of the incident electron with respect to Fermi energy in the superconductor (E_F) and L_s is the length of the stub. For $E = 0$ the phase η of the propagating electrons and holes inside the stub cancel each other as a hole follows exactly the time reversed path of an electron and α is fixed to be $= -i$. Let us define $e^{i\theta} = \eta\alpha = e^{i(2EL_s/\hbar v_F - \cos^{-1}(E/\Delta))}$ *i.e.* $\theta = 2EL_s/\hbar v_F - \cos^{-1}(E/\Delta)$ to be an energy dependent phase parameter. The constant part of the AR phase, ϕ may be set to zero without loss of generality, since it does not affect the probability.

The variation of $|t_N^{ss}|^2$ and $|t_A^{ss}|^2$ as a function of θ for various values of the parameter r' is shown in Fig. 2. Note

that when we tune the energy such that $\theta = n\pi$ (n is 0 or an integer), both the transmission probability ($|t_N^{ss}|^2$) and CAR probability ($|t_A^{ss}|^2$), are exactly $= 1/4$ for any value of the parameter r' *i.e.*, independent of the details of the $\mathbb{S}_3$ matrix. This resembles the resonance $T = 1$ (anti-resonance $T = 0$) of a standard stub geometry. Also, note that the CAR probablity is maximum at the 'resonant' value of $|t_A^{ss}|^2 = 1/4$ plotted as a function of θ (see Fig. 2(b)). On the other hand, for the normal transmission $|t_N^{ss}|^2$ plotted as a function of θ, the 'resonant value' of $1/4$ represents a maximum for $r' < -0.5$ and minimum for $r' > -0.5$ (see Fig. 2(a)). This is a very peculiar feature of this geometry. Hence $r' = -1/2$ is the crossover point where the transmission is completely flat as a function of the energy. Note also that the superconducting stub geometry does not host any true resonance ($|t_N^{ss}|^2$=1) or anti-resonance ($|t_N^{ss}|^2$=0) in sharp contrast to a normal $\mathcal{T}$-stub or the double-barrier problem.

Now the general case is when the superconductor-stub junction is not perfect and allows for both normal reflection and AR. In this case, an electron incident on the stub can enter the stub and undergo either normal reflection at the superconductor-stub junction with an amplitude r^s or it can undergo AR with an amplitude r_a^s. Hence an incident electron from reservoir-1 can either reflect back as an electron in the same wire (wire-1) or transmit as an electron into the opposite wire (wire-2) after suffering a single reflection at the superconductor-stub junction unlike the previous case where at least two bounces inside the stub were required before an incident electron entering the stub could reflect back into wire-1 as an electron or transmit into wire-2 as an electron. The coherent amplitudes for the transmission are analytically very difficult to obtain from a Feynman path approach (as was done earlier for the case of a perfect junction) *i.e.*, by summing up the coherent amplitudes for all possible paths which take an electron from the left to the right reservoir. However, for finite values of r^s, we expect the new resonance to survive as the existence of this resonance just relies on the fact that there are two channels for transport (the electron and the hole channel) and the fact that they effectively mix with one another as far as the scattering matrix describing the stub is concerned. Hence for small values of r^s, the various transmission probabilities will not change substantially with respect to the case of $r^s = 0$, close to the resonance.

Pole structure analysis:-. The subtle differences between the normal stub geometry and the superconducting stub geometry emerge once we analyze the zeros and the pole structure of the electron transmission amplitude for the two cases. For the normal stub geometry, the position of zeros and the poles of the electron transmission amplitude, t_N^{ns} as a function of the energy E (parametrised as $\theta = 2EL_s/\hbar v_F$) in the complex energy plane are given by

$$\begin{aligned}\theta_Z^n &= \theta_Z^{n0} + i0 \quad \text{(zeros)}\ ,\\ \theta_P^n &= \theta_P^{n0} - \frac{i}{2}\ln\left(\frac{1}{2r'+1}\right) \quad \text{(poles)}\ ,\end{aligned} \qquad (7)$$

where $\theta_{Z/P}^n$ represents values of E at the zeros and poles of the transmission amplitude, p is an integer and $\theta_{Z/P}^{n0} = p\pi$ represents the real part of the zero and the pole. Here the superscript n stands for normal stub and the subscript Z and P stand for the zeros and poles. In this case the zeros lie on the real energy axis, which is why the transmission function has zeros, and the poles lie in the complex plane off the real axis. But the real part of the poles coincide with the zeros. This fact can be directly related to the symmetric form of the transmission probability as a function of energy of the incident electron [12]. The stub has transmission maxima ($T^n = |t_N^{ns}|^2 = 1$) at $\theta_m^n = (2p+1)\pi/2$ where the subscript m represents maximum transmission. Expanding the transmission probability ($|t_N^{ns}|^2$) around $\theta = \theta_m^n$, one obtains the following expression,

$$T^n|_{\theta^n=\theta_m^n} = \frac{(2\Gamma)^2}{(2\Gamma)^2 + (\theta^n - \theta_m^n)^2} + \frac{\Gamma^2(\theta - \theta_m^n)^2}{(2\Gamma)^2 + (\theta^n - \theta_m^n)^2} \qquad (8)$$

where $\Gamma = t'/r$. The first term on the right hand side of Eq. 8 looks like the standard Lorentzian observed for a double barrier and the presence of the second term leads to a deviation from this shape which is expected for a Fano-type resonance [12]. But since here the real part of the zeros and poles of the transmission amplitude coincide, we get a symmetric function in energy around $\theta^n = \theta_m^n$. This is not true in general for Fano-type resonances, which could have asymmetric line shapes. Also note that the position of the transmission maxima ($T = 1$, *i.e.*, $\theta_m^n = (2p+1)\pi/2$) on the real energy axis is exactly half way between the position of the real part of the poles or the zeros (*i.e.*, $\theta_{Z/P}^{n0} = p\pi$).

Similarly, in case of the superconducting stub, we consider the energy dependent zeros and poles of the electron transmission amplitude t_N^{ss} which are given by

$$\begin{aligned}\theta_Z^s &= p\pi - \frac{i}{2}\ln\left(\frac{1}{2r'+1}\right) \quad \text{(zeros)},\\ \theta_P^s &= p\pi - i\ln\left(\frac{1}{2r'+1}\right) \quad \text{(poles)}\ ,\end{aligned} \qquad (9)$$

where $\theta_{Z/P}^s$ represents the values of $2EL_s/\hbar v_F - \cos^{-1}(E/\Delta)$ at the zeros and poles and p is an integer.

Analogous to the case of the normal stub described above, here too we find that the real part of the zeros and the poles coincide with one another, which ensures a symmetric line shape. But unlike the previous case here the zeros do not lie on the real axis. This results in the absence of zeros in the transmission probability for electrons

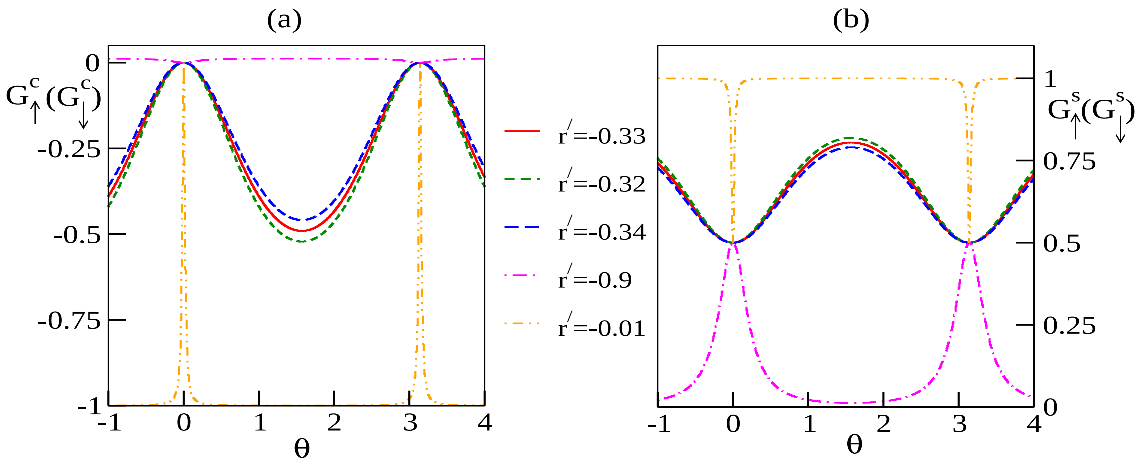

Fig. 3: (Color online)(a) Charge conductance $G^c_\uparrow(G^c_\downarrow)$ in units of e^2/h and (b) Spin conductance $G^s_\uparrow$ ($G^s_\downarrow$) in units of incident spin for spin ↑ (↓) polarised electrons plotted as a function of $\theta = 2EL_s/\hbar v_F - \cos^{-1}(E/\Delta)$ for five different values of r'.

across the superconducting stub. Also, in this case there is no transmission unity ($T^s = |t_N^{ss}|^2 = 1$) unlike the case of the normal stub. The resonances (maxima or minima depending on whether $r' < -1/2$ or $r' > -1/2$, see Fig. 2) in T^s appear at $\theta^s_p = p\pi$. Expanding the T^s around θ^s_p, we obtain the following line shape

$$T^s|_{\theta^s=\theta^s_p} = \frac{1}{4}\frac{(2\Gamma_1\Gamma_2)^2}{(2\Gamma_1\Gamma_2)^2+(\theta^s-\theta^s_p)^2} + \frac{\Gamma_2^2(\theta^s-\theta^s_p)^2}{(2\Gamma_1\Gamma_2)^2+(\theta^s-\theta^s_p)^2}\ , \tag{10}$$

where $\Gamma_1 = (2r')/(1+2r')$ and $\Gamma_2 = (1+r')/(1+2r')$. As in the case of normal stub, the first term on the right hand side of Eq. 10 represents a standard Lorentzian while the presence of the second term leads to a deviation from it. But note that the coefficient of the Lorentzian function which is unity for the normal stub becomes one quarter for the superconducting stub; this results in a resonance in transmission probability which is not unity but one quarter of unity. Also, if $r' < -0.5$ the resonance represents a transmission maxima for T^s and if $r' > -0.5$ the resonance is a minima for T^s as can be seen from the above equation (whether it is a maximum or minimum depends on whether the zero or the pole dominates). In this case also we get a perfectly symmetric line shape (see Fig. 2) since the real part of the zeros and poles of the transmission amplitude coincide with each other. Hence the main source of difference between resonances for the superconducting and the normal stub lies in the fact that for the superconducting stub the zeros of the complex transmission amplitude lie in the complex plane off the real θ axis while for the normal stub they lie on the real θ axis itself. This explains the absence of zeros in the transmission amplitude in the superconducting stub geometry.

Charge and spin currents:-. As an application of the above geometry, we note that this geometry can be used to produce pure SC . A similar proposal for the production of pure SC involving a NSN junction was also discussed in the past by the present authors in Ref. [10] as mentioned before. In the present case, the pure SC is shown to be coupled to the new resonance discussed above. The main point to note here is that when an electron is incident on the stub, then if the amplitudes for the normal transmission t_N^{ss} and CAR t_A^{ss} are identical, then the probabilities for an incident electron to transmit as an electron or a hole across the stub are identical, and effectively the transmitted charge current will be zero. Furthermore, if the incident electron is spin polarized, then the amplitudes for the transmitted electron (t_N^{ss}) and holes (t_A^{ss}) will also have the same spin polarization as long as the superconductor at the junction is a singlet superconductor. This results in pure spin current with zero net charge current.

Thus, for spin polarized electrons when the superconducting stub is tuned to resonance, *i.e.*, $|t_N^{ss}|^2 = |t_A^{ss}|^2 = \frac{1}{4}$, the outcome will be resonant production of pure SC . In this resonant situation 25% of the incident spin-up electrons get transmitted through the stub via the normal transmission process and 25% get converted to spin-up holes via the CAR process, as they pass through the $\mathcal{T}$-stub. Hence the transmitted charge across the $\mathcal{T}$-stub is zero on the average, but there is pure SC flowing out of the system. At zero temperature limit the linear response charge conductance of incident spin polarized electrons is given by $G_{\uparrow(\downarrow)} = e^2/h(|t_A^{ss}|^2 - |t_N^{ss}|^2)$ and the spin conductance $G^s_{\uparrow(\downarrow)} \propto (|t_A^{ss}|^2 + |t_N^{ss}|^2)$. These are depicted in Fig.3. The important point to note in this geometry is that the maxima in the SC is accompanied by zero charge current. At resonance, the charge of the incident electron is absorbed by the superconductor, and no charge is either

transmitted or reflected from the $\mathcal{T}$-stub on the average. On the other hand, in units of the original spin of the electron, probablistically on an average, half of the spin is transmitted and the remaining half is reflected back.

Conclusions and Discussion. – In conclusion, we have shown that the superconducting stub geometry is very different from the normal stub geometry in terms of its resonances or anti-resonances. The presence of both electron and hole waves in the stub leads to an unusual interference pattern, which causes resonances at a non-unimodular value of the transmission. We perform a comprehensive analysis of the analytic structure of the transmission amplitude for the electron in the complex energy plane for the proposed geometry. We have also discussed a possible application of the geometry in resonant production of pure SC by allowing incidence of spin polarised electrons on the $\mathcal{T}$-stub geometry. At 'resonance', we find that the charge transport across the stub cancels out to zero, because normal transmission and CAR probability are both equal to 1/4, whereas spin transport is non-zero and becomes 1/2 in the unit of spin that is transmitted at resonance. It is also worth pointing out that this new resonance is distinct from the standard resonances or anti-resonances at $T = 1$ or $T = 0$ from the point of view of noise associated with these resonances. As long as $T = 1$ or $T = 0$, the current fluctuations are identically zero (*i.e.*, we have zero noise) at zero temperature [16]. But these new resonances allow for finite noise at resonance even at zero temperature. Although our analysis is strictly true for a single channel quantum wire, we expect the new resonance to survive for a multi-channel wire, provided the inter channel mixing is suppressed at the junction.

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